

Extended Chaplygin Gas in Horava-Lifshitz Gravity

B. Pourhassan^a

^a*School of Physics, Damghan University, Damghan, Iran*

E-mail: b.pourhassan@du.ac.ir

ABSTRACT: In this paper, we investigate cosmological models of the extended Chaplygin gas in a universe governed by Horava-Lifshitz gravity. The equation of state for an extended Chaplygin gas is a $(n + 2)$ -variable equation determined by A_n , α , and B . In this work, we are interested to the case of second order ($n = 2$) equation of state which recovers quadratic barotropic equation of state. In that case there are four free parameters. We solve conservation equation approximately and obtain energy density in terms of scale factor with mentioned free parameters. Under some assumptions we relate free parameters to each other to have only one free independent parameter (A_2). It help us to obtain explicit expression for energy density in terms of scale factor. The allowed values of the second order extended Chaplygin gas parameter is fixed using the recent astrophysical and cosmological observational data. Thermodynamics of the model investigated based on the first and second law of thermodynamics.

KEYWORDS: Dark Energy; Horava-Lifshitz; Chaplygin Gas.

Contents

1	Introduction	1
2	Horava-Lifshitz cosmology	3
3	Extended Chaplygin gas	4
4	Observational constraints	6
5	Cosmological parameters	7
6	Thermodynamics	10
7	Conclusion	11

1 Introduction

Cosmological observations verify the accelerated expansion of the universe, including the deceleration to the accelerated phase transition [1–3]. Such accelerated expansion can be described by dark energy models. One of the primordial candidates of dark energy is the cosmological constant, which is not dynamical model, so there are some alternative models such as quintessence model [4–9], phantom model [10–16], or quintom model [17–19]. Holographic model of dark energy is another interesting description of the dark energy [20–23]. In order to construct dynamical version of cosmological constant model, one can introduce interaction term between dark matter and dark energy [24–33].

There are also other interesting models to describe the dark energy such as Chaplygin gas [34, 35], which emerged initially in cosmology from string theory point of view [36, 37], which are based on Chaplygin gas (CG) equation of state and developed to the generalized Chaplygin gas (GCG) [38]. It is also possible to enter the presence of viscosity in GCG [39–43]. Then, GCG was extended to the modified Chaplygin gas (MCG) [44]. Recently, viscous MCG is also suggested and studied [45, 46]. A further extension of CG model is called modified cosmic Chaplygin gas (MCCG) which was proposed recently [47, 48].

The MCG equation of state (EoS) has two parts, the first term gives an ordinary fluid obeying a linear barotropic EoS, and the second term relates pressure to some power of the the inverse of energy density. So, one essentially dealing with a two fluid model. However, it is possible to consider barotropic fluid with quadratic EoS or even with higher orders EoS [49–51]. Therefore, it is interesting to extend MCG EoS which recovers at least barotropic fluid with quadratic EoS, and is called extended Chaplygin gas (ECG) [52–55].

On the other hand, Horava-Lifshitz (HL) gravity appears to be an attractive model to achieve a complete quantum gravitational theory [56]. As we know, there are several open

issues in HL gravity including the classical and quantum instability of the scalar modes which can be solved under some assumptions like consideration of projectable HL gravity and principle of detailed balance. In that case, HL gravity in the presence of a scalar field studied by the Ref. [57] where the effect of detailed balance conditions investigated. Moreover it is mentioned that HL theory of gravity is not exactly consistent with general theory of relativity at low energy [58].

HL gravity has some application in the black hole properties [59–64], the thermodynamic properties [65–69], the dark energy phenomenology [70–72], etc. Additionally, application of HL gravity as a cosmological framework gives rise to HL cosmology [73, 74].

In that case, HL cosmology with GCG has been studied by the Ref. [75]. Also, MCG in HL gravity and observational constraints has been studied by the Refs. [76, 77] with possibility of extension to the case of varying G and Λ [78]. Now, we would like to investigate ECG in HL gravity. The equation of state for a ECG is a $n + 2$ -variable equation determined by A_n , α and B . We assume second order EoS which recovers quadratic barotropic EoS, and by using special assumptions, reduce number of free parameters to one. Therefore, we have only an independent parameter. The allowed values of mentioned EoS parameter are fixed by using the recent astrophysical and cosmological observational data such as $H(z)$ analysis.

On the other hand, the temperature behavior and the thermodynamic stability of the generalized Chaplygin gas has been studied by the Ref. [79], and it is found that the generalized Chaplygin gas cools down through the expansion without facing any critical point or phase transition. Also, thermodynamics of the generalized Chaplygin gas has been investigated by introducing the integrability condition, and thermodynamic quantities have been derived as functions of either volume and temperature [80]. Validity of the generalized second law of gravitational thermodynamics in a non-flat Friedmann-Robertson-Walker (FRW) universe and an expanding Gödel-type universe containing the generalized Chaplygin gas confirmed by the Refs. [81] and [82] respectively. In the extension of the Ref. [79], the similar work performed for the case of the modified Chaplygin gas [83] and the same result obtained. More discussion on thermodynamical behavior of the modified Chaplygin gas found in the Ref. [84]. Also, Ref. [85] developed the Ref. [80] to the case of the modified Chaplygin gas. Validity of the generalized second law of thermodynamics in the presence of the modified Chaplygin gas investigated by the Ref. [86] and observed that the generalized second law of thermodynamics always satisfied for the modified Chaplygin gas model. The generalized second law of thermodynamics in the brane-world scenario including the modified Chaplygin gas verified on the apparent horizon in late time by the Ref. [87].

Already, the thermodynamics in HL cosmology have been investigated and validity of the generalized second law of thermodynamics verified [69]. So, there is no thermodynamical study of the ECG in HL cosmology, which is also another subject of this paper.

The paper is organized as follows. In the next section we give a brief review of HL cosmology, then in section 3 we introduce the ECG. In section 4 we use observational data to constrain the free parameters of the model. In section 5 we investigate some cosmological parameters and in section 6 we study thermodynamical aspects of the model and finally in section 7 we give conclusions and suggestions for future works.

2 Horava-Lifshitz cosmology

HL gravity described by the following metric [75],

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (2.1)$$

where N and N^i are the lapse and shift functions which are used in general relativity in order to split the space-time dimensions. Using the projectable version of HL gravity [88] with the detailed balanced principle [89] one can write the gravity action of HL theory as follow,

$$S = \int dt dx^3 \sqrt{g} N [\tilde{\mathcal{L}}_0 + \mathcal{L}_0 + \mathcal{L}_1], \quad (2.2)$$

where,

$$\begin{aligned} \tilde{\mathcal{L}}_0 &= \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2), \\ \mathcal{L}_0 &= -\frac{\kappa^2}{2\omega^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu}{2\omega^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R_k^l - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij}, \\ \mathcal{L}_1 &= \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right), \end{aligned} \quad (2.3)$$

where κ , λ , μ and ω are constant parameters, Λ is a positive constant, which as usual is related to the cosmological constant in the IR limit, R_{ij} and R are Ricci tensor and Ricci scalar respectively. Also, the Cotton tensor is defined as follow,

$$C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k (R_i^j - \frac{1}{4} R \delta_i^j), \quad (2.4)$$

also the extrinsic curvature is defined as,

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i). \quad (2.5)$$

It is usual to use FRW metric with $N = 1$ and $N^i = 0$ to obtain the following Friedmann equations,

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)} \rho - \frac{\kappa^4 \mu^2}{8(3\lambda-1)^2} \left(\frac{\Lambda k}{a^2} - \frac{k^2}{2a^4} - \frac{1}{2} \Lambda^2 \right), \quad (2.6)$$

and,

$$\dot{H} + \frac{3}{2} H^2 = -\frac{\kappa^2}{4(3\lambda-1)} p - \frac{\kappa^4 \mu^2}{16(3\lambda-1)^2} \left(\frac{\Lambda k}{a^2} + \frac{k^2}{2a^4} - \frac{3}{2} \Lambda^2 \right), \quad (2.7)$$

where $H = \dot{a}/a$ and a are Hubble parameter and scale factor respectively, k is curvature constant corresponding to open ($k < 0$), flat ($k = 0$), and closed ($k > 0$) universe. Also, p and ρ are corresponding to total pressure and energy density which contain radiation, dark matter and dark energy.

Using the usual notifications,

$$G_{cosmo} = \frac{\kappa^2}{16\pi(3\lambda-1)}, \quad (2.8)$$

$$G_{grav} = \frac{\kappa^2}{32\pi}, \quad (2.9)$$

and,

$$\frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2} = 1, \quad (2.10)$$

the conservation equations are,

$$\dot{\rho}_r + 3H(p_r + \rho_r) = 0, \quad (2.11)$$

$$\dot{\rho}_b + 3H\rho_b = 0, \quad (2.12)$$

and,

$$\dot{\rho}_c + 3H(p_c + \rho_c) = 0, \quad (2.13)$$

where p_r and ρ_r are pressure and energy density of radiation, ρ_b is energy density of baryonic matter which is pressureless, and p_c and ρ_c are pressure and energy density of the extended Chaplygin gas which is the unification of the dark matter and dark energy and discussed in the next section.

3 Extended Chaplygin gas

The extended Chaplygin gas EoS given by [52, 53],

$$p_c = \sum_{n=1}^{\infty} A_n \rho_c^n - \frac{B}{\rho_c^\alpha}, \quad (3.1)$$

where A_n , B , α and n are constants, so we have generally $n + 2$ free parameters. Note that in the case $n = 1$ the above expressions recovers the standard MCG. In this paper, we are interested to the case of second order EoS ($n = 2$) which recovers quadratic barotropic EoS. In that case the EoS given by (3.1) reduced to the following expression,

$$p_c = A_1 \rho_c + A_2 \rho_c^2 - \frac{B}{\rho_c^\alpha}. \quad (3.2)$$

Assuming $A_2 = 0$ gives MCG [76], while $A_2 = A_1 = 0$ gives GCG [75] where A_1 , B , and α are positive constants with $0 < \alpha \leq 1$. Therefore, we have four free parameters in our model.

Using the conservation equation (2.13) and ECG equation of state (3.2) we can obtain,

$$\rho_c = \left[\frac{B}{1 + A_1} + \frac{c}{a^{3(1+A_1)(1+\alpha)}} e^{-(1+\alpha)(1+A_1)f(\rho_c)} \right]^{\frac{1}{1+\alpha}} \quad (3.3)$$

where $c = (1 + A_1)^{-1}$ and

$$\begin{aligned} f(\rho_c) &= \frac{A_2 \rho_c}{(1 + A_1)^2} - \frac{B A_2 \rho_c}{(1 + \alpha)(1 + A_1)^2((1 + A_1)\rho_c^{1+\alpha} - B)} \\ &+ A_2 \int \frac{B(2 + \alpha)}{(1 + \alpha)(1 + A_1)^2((1 + A_1)\rho_c^{1+\alpha} - B)} d\rho_c. \end{aligned} \quad (3.4)$$

In the case of $A_2 = 0$, then $f(\rho_c) = 0$, so we recover the result which is obtained by MCG. In order to isolate ρ_c completely and find correct relation of ρ_c in terms of a , we consider special case of $\alpha \ll 1$, in that case integration of last term in (3.4) is solved analytically and we find,

$$\int \frac{B(2+\alpha)}{(1+\alpha)(1+A_1)^2((1+A_1)\rho_c^{1+\alpha} - B)} d\rho_c|_{\alpha \rightarrow 0} = \frac{2B}{(1+A_1)^3} \ln((1+A_1)\rho_c - B). \quad (3.5)$$

Asymptotic behavior (early universe where ρ_c is very big) together $a \ll 1$ allow us to choose,

$$ce^{-(1+\alpha)(1+A_1)f(\rho_c)} \approx 1 + C(A_2)\rho_c^{-(1+\alpha)}, \quad (3.6)$$

where $C(A_2)$ is a constant depend on A_2 which is zero at $A_2 = 0$. Under these assumptions one can obtain,

$$\rho_c^{1+\alpha} = \frac{1}{2} \left(\frac{B}{1+A_1} + \frac{1}{a^{3(1+A_1)(1+\alpha)}} \right) + \frac{1}{2} \sqrt{\left(\frac{B}{1+A_1} + \frac{1}{a^{3(1+A_1)(1+\alpha)}} \right)^2 + \frac{4C(A_2)}{a^{3(1+A_1)(1+\alpha)}}}, \quad (3.7)$$

this is appropriate solution only at the early universe where ρ_c is very big. This is reasonable because terms of ρ_c^n in ECG are more important at the early universe. If we use relation (3.7) for the late time, then resulting cosmological parameter are far from observation which will be illustrated graphically in the next sections. So it is desirable to find a solution which yields to good results for both early and late time.

There is also alternative way to obtain exact solution. We assume the following conditions,

$$\begin{aligned} \alpha &= 1, \\ A_1 &= A_2 - 1, \\ B &= 2A_2. \end{aligned} \quad (3.8)$$

Therefore, the only free parameter of the model is A_2 , and we can solve the conservation equation (2.13) to obtain the following relation,

$$\rho_c = c_2 \frac{(1+x+\sqrt{5x-1})}{(x-1)}, \quad (3.9)$$

with $x = c_1 e^{3\pi} a^{30A_2}$, where c_1 is constant of integration and c_2 comes from the fact that the equation (3.2) is dimensionless. The density is physically defined only for $x > 1$ (the density is negative for $x < 1$). In this model, the universe starts at $x = 1$ with an infinite density, then the density decreases and finally reaches an asymptotic value for $x \rightarrow +\infty$.

In order to write solution (3.9), we used $\tan^{-1}(\rho_c + 1) \approx \pi/2$ approximation, which is valid when $\rho_c \gg 1$ corresponding to the early universe. However, our solution will be valid at all time and our approximate solution is very close to the late time behavior with $\rho_c \ll 1$. This is due to the fact that $\tan^{-1}(\rho_c + 1) \approx \pi/4$, for $\rho_c \ll 1$. Therefore, we will use energy density (3.9) instead of (3.7) which is only appropriate for the early universe with $\rho_c \gg 1$. Using numerical analysis will show that energy density (3.9) yields to better results.

The equation (3.2) is implicitly normalized by the cosmological density ρ_L , i.e. by the

asymptotic value of ρ_c for $a \rightarrow +\infty$. Indeed, we can see that for $\rho_c = 1$ we get $p_c = A_1 + A_2 - B = -1$ which corresponds to the equation of state $p = -\rho$ expected for $a \rightarrow +\infty$. According to this normalization, we must take $c_2 = 1$. Then, we have to determine c_1 . The present density and the cosmological density are related to each other by $\rho_0 = 1.31\rho_L$. This means that with the previous normalization we should take $\rho_c = 1.31$ when $a = 1$. This gives $c_1 e^{(3\pi)} = 65$, so we can write $x = 65a^{(30A_2)}$.

Similarly to the detailed balance case, in the IR with $\lambda = 1$, $G_{cosmo} = G_{grav} \equiv G$. So, using (2.8), (2.9) and (2.10) one can rewrite Friedmann equations (2.6) and (2.7) as follows,

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{2} - \frac{k}{a^2} + \frac{k^2}{2\Lambda a^4}, \quad (3.10)$$

and,

$$\dot{H} + \frac{3}{2}H^2 = -4\pi Gp + \frac{3\Lambda}{4} - \frac{k}{2a^2} - \frac{k^2}{4\Lambda a^4}, \quad (3.11)$$

where,

$$\rho = \rho_r + \rho_b + \rho_c, \quad (3.12)$$

is total energy density including radiation, baryonic matter, and extended Chaplygin gas respectively. Therefore,

$$p = \frac{1}{3}\rho_r + (A_2 - 1)\rho_c + A_2\rho_c^2 - \frac{2A_2}{\rho_c}, \quad (3.13)$$

is the total pressure with the fact that $\omega_b = p_b/\rho_b = 0$, $\omega_r = p_r/\rho_r = 1/3$ and $\omega_c = p_c/\rho_c$. Finally it is useful to define the following relations,

$$\Omega_i = \frac{8\pi G}{3H^2}\rho_i, \quad \Omega_k = -\frac{k}{a^2 H^2}, \quad \Omega_0 = \frac{\Lambda}{2H_0^2}, \quad (3.14)$$

with $i = b, r, c$.

4 Observational constraints

In this section, we use $H(z)$ data to fix some model parameters. In order to use observational data it is useful to rewrite equations in terms of redshift. In that case, using the following relation,

$$a = \frac{a_0}{1+z}, \quad (4.1)$$

together with (2.13) and (3.9) in the equation (3.10) we can obtain Hubble expansion parameter in terms of redshift as follow,

$$E^2(z) = \Omega_{r0}(1+z)^4 + \Omega_{b0}(1+z)^3 + \Omega_{c0}F(z) + \Omega_0 + \Omega_{k0}(1+z)^2 + \frac{\Omega_{k0}^2}{4\Omega_0}(1+z)^4, \quad (4.2)$$

where,

$$E(z) \equiv \frac{H(z)}{H_0}, \quad (4.3)$$

and,

$$F(z) = \frac{(1 + 65(1+z)^{-30A_2} + \sqrt{325(1+z)^{-30A_2} - 1})}{(65(1+z)^{-30A_2} - 1)}, \quad (4.4)$$

with the current value of the Hubble expansion parameter H_0 . An important free parameter is A_2 which should fixed using observational data. Moreover, present day ($z = 0$) radiation, baryon, ECG, cosmological constant, and curvature energy densities are denoted by Ω_{r0} , Ω_{b0} , Ω_{c0} , Ω_0 , Ω_{k0} respectively which satisfy the following equation,

$$1 = \Omega_{r0} + \Omega_{b0} + \Omega_{c0} + \Omega_0 + \Omega_{k0} + \frac{\Omega_{k0}^2}{4\Omega_0}. \quad (4.5)$$

It is obvious that the value of A_2 is not important at present stage and, as expected, it is important at the early universe. The last term in the equation (4.5) corresponds to the dark radiation, which is a characteristic feature of the HL theory of gravity and restricted as follow [90],

$$\frac{\Omega_{k0}^2}{4\Omega_0} = 0.135\Delta N_\nu \Omega_{r0}, \quad (4.6)$$

where ΔN_ν represents the effective neutrino species with the following bound [90],

$$-1.7 \leq \Delta N_\nu \leq 2. \quad (4.7)$$

However, we restrict ourself to the case of $0 \leq \Delta N_\nu \leq 2$ [76]. Using the equation (4.6) in the relation (4.5) it is easy to find [76],

$$\Omega_0 = 1 - \Omega_{b0} - \Omega_{c0} - (1 - 0.135\Delta N_\nu)\Omega_{r0} - 0.73\sqrt{\Delta N_\nu(\Omega_{r0} - (\Omega_{b0} + \Omega_{c0})\Omega_{r0} - \Omega_{r0}^2)}. \quad (4.8)$$

Then, equation (4.6) gives,

$$\Omega_{k0} = \sqrt{0.54\Omega_0\Delta N_\nu\Omega_{r0}}. \quad (4.9)$$

So, in numerical analysis, similar to the Ref. [76], we choose $H_0 = 71.4 \text{ Km/s/Mpc}$, $\Omega_{b0} = 0.04$, $\Omega_{r0} = 8.14 \times 10^{-5}$, and $\Omega_{c0} = 0.951$. These yield to $0.0080 \leq \Omega_0 \leq 0.0089$ and $0 \leq \Omega_{k0} \leq 0.00084$. In the Fig. 1 we represent our numerical results together experimental data [91]. In order to have more agreement with seven year WMAP data [92] we suggests $0.1 \leq A_2 < 0.15$ for all $0 \leq \Delta N_\nu \leq 2$. Also, dashed and dotted lines of $H(z)$ show that approximate solution given by (3.9) may lead to unexpected results.

5 Cosmological parameters

Using the obtained results for model parameters we investigate behavior of some important cosmological parameters. The first quantity is the effective EoS parameter given by,

$$\omega_{eff} = \frac{p_{eff}}{\rho_{eff}}, \quad (5.1)$$

where,

$$p_{eff} = p + \frac{2}{\kappa^2} \left[\frac{k^2}{\Lambda a^4} - 3\Lambda \right], \quad (5.2)$$

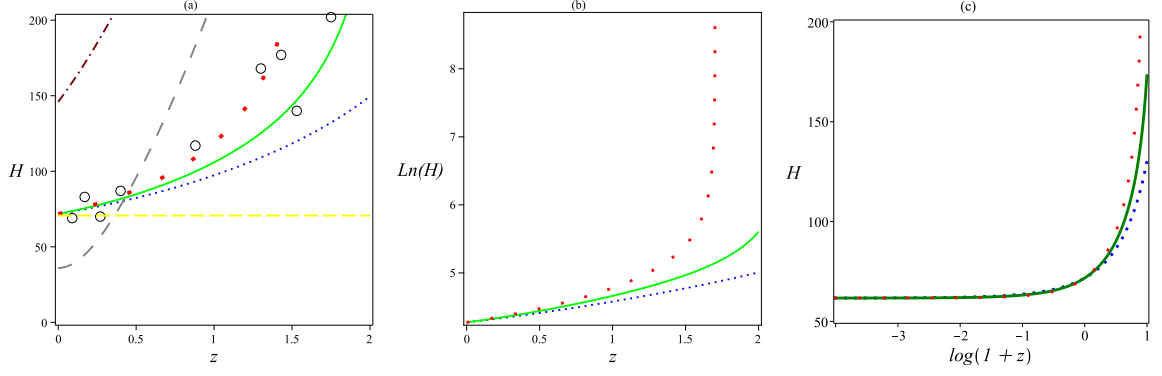


Figure 1. (a) Hubble expansion parameter in terms of redshift for the exact solution given by (3.9) with $A_2 = 0.1$ (dotted blue), $A_2 = 0.12$ (solid green), $A_2 = 0.14$ (space dotted red). Dashed yellow ($k = 0$), space dashed Gray ($k = 1$) and dash dotted ($k = -1$) lines are corresponding to approximate solution given by (3.7) with $B = 0.1$, $C(A_2) = 1$ and $A_1 = -0.9$. Circles represent $H(z)$ data. (b) Logarithmic Hubble parameter with $A_2 = 0.1$ (dotted blue), $A_2 = 0.12$ (solid green), $A_2 = 0.14$ (space dotted red). (c) Log scale of Hubble parameter for $1 + z$ with $A_2 = 0.1$ (dotted blue), $A_2 = 0.12$ (solid green), $A_2 = 0.14$ (space dotted red).

and,

$$\rho_{eff} = \rho + \frac{2}{\kappa^2} \left[\frac{3k^2}{\Lambda a^4} + 3\Lambda \right], \quad (5.3)$$

with the p and ρ given by the equations (3.12) and (3.13). In the Fig. 2 we can see behavior of the EoS parameter for open, close and flat universe. In the case of flat universe we see $\omega_{eff} \rightarrow -1$ while in the cases of closed or open universes, the present value of the effective EoS parameter is about -0.7 which admits accelerating universe in agreement with observational data [93].

In order to have a fair evolution of the model it is desirable to compare the equation of state parameter with the current bounds on $\omega(a) = \frac{\omega_{eff}}{\Omega_c}$. As we know, there are several ways to parameterize the equation of state parameter for example Chevallier-Polarski-Linder (CPL) parameterizations [94, 95],

$$\omega(a) = \omega_0 + \omega_a(1 - a) = \omega_0 + \omega_a \frac{z}{1 + z}, \quad (5.4)$$

where ω_0 corresponds to the present day value and [97]

$$\omega_a = (d\omega(a)/dz)|_{z=0} = (-d\omega(a)/da)|_{a=1}. \quad (5.5)$$

The current constraints for the CPL parameters are $\omega_0 = -1.11 \pm 0.17$ and $\omega_a = 0.34 \pm 0.60$ [96]. We can summarize our results using present value $\Omega_{c0} = 0.951$ [76] in the table 1 and see agreement with current data.

A_2	ω_0	ω_a
0.11	-1.02660	0.05699
0.13	-1.02300	0.07297
0.15	-1.01960	0.09158
0.17	-1.01619	0.11282
0.19	-1.01270	0.13680

Table 1. Values of ω_0 and ω_a of our model with $\Lambda = \kappa = 1, k = 0$ and different values of A_2 , in agreement with $\omega_0 = -1.11 \pm 0.17$ and $\omega_a = 0.34 \pm 0.60$ [96].

We can also investigate stability of the model using sound speed which is given by,

$$C_s^2 = \frac{dp}{d\rho}, \quad (5.6)$$

in which, the model is stable if $C_s^2 \geq 0$. In the Fig. 3 we can see that the model is completely stable for open and closed universes, while there are some instabilities in the flat universe for $0 \leq A_2 \leq 0.2$.

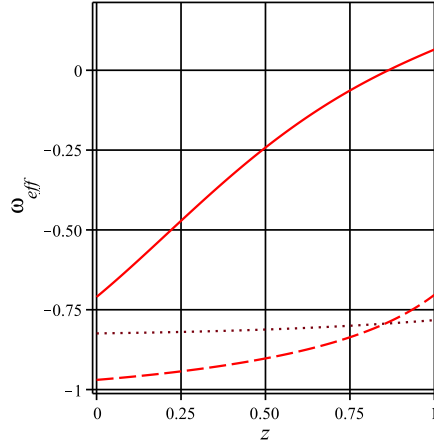


Figure 2. EoS in terms of redshift for the exact solution given by (3.9) with $A_2 = 0.15$, open and closed universe denoted by solid line and flat universe denoted by dashed line. Dotted line represent approximate solution given by (3.7) with $B = 0.1$, $C(A_2) = 1$ and $A_1 = -0.9$ for open and closed universe.

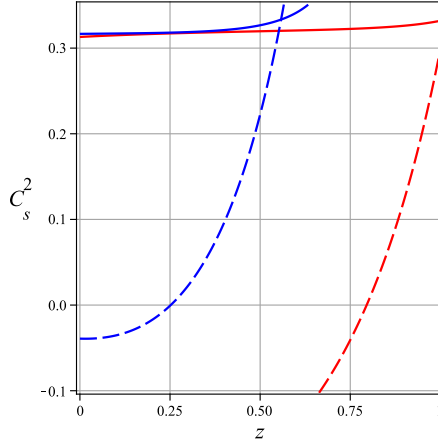


Figure 3. Squared sound speed in terms of redshift for $A_2 = 0.15$ (red line), $A_2 = 0.2$ (blue line). Open and closed universe denoted by solid line and flat universe denoted by dashed line.

6 Thermodynamics

It is important to investigate thermodynamic properties of the model which may lead to study the generalized second law of thermodynamic. In order to do that we consider the universe as a thermodynamical system with the apparent horizon surface being its boundary. The apparent horizon given by [69],

$$r_A = \left(H^2 + \frac{k}{a^2} \right)^{-\frac{1}{2}}. \quad (6.1)$$

Then, the temperature and the entropy are given by the following expressions respectively,

$$T = \frac{1}{2\pi r_A}, \quad (6.2)$$

and,

$$S = \frac{\kappa^2}{32\Lambda G^2} \left[\Lambda r_A^2 + 2k \ln(\sqrt{\Lambda} r_A) \right]. \quad (6.3)$$

We should have mentioned that the first term corresponds to the general relativity, while the second one arises from HL gravity. Using the equation (3.10) we can obtain,

$$T = \frac{\sqrt{\frac{8\pi G}{3}\rho + \frac{\Lambda}{2} + \frac{k^2}{2\Lambda a^4}}}{2\pi}, \quad (6.4)$$

and,

$$S = \frac{\kappa^2}{32\Lambda G^2} \left[\frac{\Lambda}{\frac{8\pi G}{3}\rho + \frac{\Lambda}{2} + \frac{k^2}{2\Lambda a^4}} + 2k \ln \left(\sqrt{\frac{\Lambda}{\frac{8\pi G}{3}\rho + \frac{\Lambda}{2} + \frac{k^2}{2\Lambda a^4}}} \right) \right]. \quad (6.5)$$

As we can see from the equation (6.3), the apparent horizon is a function of Hubble parameter, so it is a function of time. Therefore, dr_A is corresponding to dV . In that case, the first law of thermodynamics [69] given by,

$$TdS = dE + pdV, \quad (6.6)$$

where $V = \frac{4}{3}\pi r_A^3$ is the volume of the system bounded by the apparent horizon, so dV denotes volume-change and $dE = \rho dV$, where $E = \frac{4}{3}\pi r_A^3 \rho$ is energy. So, one can use (2.11), (2.12), and (2.13) in the equation (6.6), and obtain,

$$\frac{dS_2}{dt} = \frac{4\pi}{T}(1+\omega)\rho r_A^2 \left(\frac{dr_A}{dt} - Hr_A \right). \quad (6.7)$$

where S_2 is the entropy given by the first law of thermodynamics (6.6). Using the relation (6.2) gives,

$$\frac{dS_2}{dt} = 2(1+\omega)\rho r_A \left(\frac{dr_A}{dt} - Hr_A \right). \quad (6.8)$$

In that case one can relate the horizon entropy to r_A . It has been constructed using the relation (6.3) [60, 66, 67]. Differentiating (6.3) we have,

$$\frac{dS_1}{dt} = \frac{\kappa^2}{16\Lambda G^2} \left[\Lambda r_A + \frac{k}{r_A} \right] \frac{dr_A}{dt}, \quad (6.9)$$

where S_1 is the entropy given by the apparent horizon. As expected, the first term is useful while the second term coming from HL gravity. Now, we can calculate total entropy change,

$$\frac{dS_{tot}}{dt} = \frac{dS_1}{dt} + \frac{dS_2}{dt}. \quad (6.10)$$

It is easy to check that $\frac{dS_{tot}}{dt} > 0$, at least for flat and closed universe, in agreement with the results reported by Ref. [69]. In the case of open universe ($k = -1$), it should be $r_A^2 \geq \frac{1}{\Lambda}$, to have $\frac{dS_1}{dt} \geq 0$.

We have the following conditions to have conserved entropy ($\frac{dS_{tot}}{dt} = 0$),

$$\begin{aligned} \frac{dr_A}{dt} &= Hr_A, \\ r_A^2 &= -\frac{k}{\Lambda}. \end{aligned} \quad (6.11)$$

Both conditions are satisfied simultaneously, only in the case of varying Λ (slowly) as follow,

$$\Lambda = \pm C e^{-2 \int H dt}, \quad (6.12)$$

for the closed and open universe. Otherwise, in the case of constant Λ , total entropy is not conserved.

7 Conclusion

In this paper, we have considered the extended Chaplygin gas in Horava-Lifshitz gravity. First of all we reviewed the HL cosmology and then solved conservation equation for the special case of the ECG at the second order. Also, we solved the general case approximately and obtained energy density in terms of scale factor which allow us to investigate Hubble expansion parameter in terms of redshift. We have considered some assumptions which reduced free parameters of the ECG to one. We used $H(z)$ data to constrain this parameter in HL cosmology. $H(z)$ observations suggest that $0.13 \leq A_2 \leq 0.25$, which means $0.26 \leq$

$B \leq 0.5$ and $-0.87 \leq A_1 \leq -0.75$. Using the obtained parameters of the model we have studied evolution of EoS parameter and found that, for the both cases $k < 0$ and $k > 0$, the present value of the effective EoS parameter is about -0.7, while it yields to 1/3 at high redshift. On the other hand, for the flat universe ($k = 0$) we found $\omega_{eff} \rightarrow -1$.

Stability of the model is also investigated by using the squared sound speed and found that the model is completely stable for the open and closed universes, while there are some instabilities in flat universe for the case of $0.1 \leq A_2 \leq 0.2$. It seems $A_2 = 0.15$ is the best fitted value.

Finally, we have investigated thermodynamic properties of the model. We found that the generalized second law of thermodynamics is valid for a flat or closed universe. We found that the total entropy may be conserved in the case of varying Λ .

In this work we have neglected the quantum effects near the black hole horizon, so it would be interesting to consider such effects for example using logarithmic corrected entropy [98]. Finally an interesting work may be the consideration of unified Chaplygin gas with the following equation of state,

$$p = \sum_{i \in R} A_i \rho^i, \quad (7.1)$$

where i may be positive, negative, integer and non-integer number. It covers all kinds of Chaplygin gas EoS and will be investigate as separate work.

Acknowledgments

Author would like to thank P. H. Chavanis and E.O. Kahya for useful comments and discussions. Also special thanks to Hoda Farahani for reading manuscript and helpful discussions.

References

- [1] A. G. Riess *et al.* [Supernova Search Team Collaboration], *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, Astron. J. **116**, 1009 (1998), [[arXiv:astro-ph/9805201](#)].
- [2] A.G. Riess *et al.* [Supernova Search Team Collaboration], *Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, Astron. J. **607**, 665 (2004), [[arXiv:astro-ph/0402512](#)].
- [3] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Measurements of Omega and Lambda from 42 high redshift supernovae*, Astrophys. J. **517**, 565 (1999), [[arXiv:astro-ph/9812133](#)].
- [4] B. Ratra and P. J. E. Peebles, *Cosmological Consequences of a Rolling Homogeneous Scalar Field*, Phys. Rev. D **37**, 3406 (1988).
- [5] C. Wetterich, *Cosmology and the Fate of Dilatation Symmetry*, Nucl. Phys. B **302**, 668 (1988).

- [6] A. R. Liddle and R. J. Scherrer, *A Classification of scalar field potentials with cosmological scaling solutions*, Phys. Rev. D **59**, 023509 (1999), [[arXiv:astro-ph/9809272](#)].
- [7] Z. -K. Guo, N. Ohta and Y. -Z. Zhang, *Parametrizations of the dark energy density and scalar potentials*, Mod. Phys. Lett. A **22**, 883 (2007), [[arXiv:astro-ph/0603109](#)].
- [8] M. Khurshudyan, E. Chubaryan, B. Pourhassan, *Interacting Quintessence Models of Dark Energy*, Int. J. Theor. Phys. **53**, 2370 (2014), [[arXiv:1402.2385](#) [gr-qc]].
- [9] S. Dutta, E. N. Saridakis and R. J. Scherrer, *Dark energy from a quintessence (phantom) field rolling near potential minimum (maximum)*, Phys. Rev. D **79**, 103005 (2009), [[arXiv:0903.3412](#)].
- [10] R. R. Caldwell, *A Phantom menace?*, Phys. Lett. B **545**, 23 (2002), [[arXiv:astro-ph/9908168](#)].
- [11] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, *Phantom energy and cosmic doomsday*, Phys. Rev. Lett. **91**, 071301 (2003), [[arXiv:astro-ph/0302506](#)].
- [12] S. Nojiri and S. D. Odintsov, *Quantum de Sitter cosmology and phantom matter*, Phys. Lett. B **562**, 147 (2003), [[arXiv:hep-th/0303117](#)].
- [13] V. K. Onemli and R. P. Woodard, *Quantum effects can render $w < -1$ on cosmological scales*, Phys. Rev. D **70**, 107301 (2004), [[arXiv:gr-qc/0406098](#)].
- [14] E. N. Saridakis, *Theoretical Limits on the Equation-of-State Parameter of Phantom Cosmology*, Phys. Lett. B **676**, 7 (2009), [[arXiv:0811.1333](#)].
- [15] E. N. Saridakis, *Phantom evolution in power-law potentials*, Nucl. Phys. B **819**, 116 (2009), [[arXiv:0902.3978](#)].
- [16] G. Gupta, E. N. Saridakis and A. A. Sen, *Non-minimal quintessence and phantom with nearly flat potentials*, Phys. Rev. D **79**, 123013 (2009), [[arXiv:0905.2348](#)].
- [17] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, *Cosmological evolution of a quintom model of dark energy*, Phys. Lett. B **608**, 177 (2005), [[arXiv:astro-ph/0410654](#)].
- [18] W. Zhao, *Quintom models with an equation of state crossing -1*, Phys. Rev. D **73**, 123509 (2006), [[arXiv:astro-ph/0604460](#)].
- [19] Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, *Quintom Cosmology: Theoretical implications and observations*, Phys. Rept. **493**, 1 (2010), [[arXiv:0909.2776](#)].
- [20] H. Li, Z. K. Guo and Y. Z. Zhang, *A Tracker Solution for a Holographic Dark Energy Model*, Int. J. Mod. Phys. D **15**, 869 (2006), [[arXiv:astro-ph/0602521](#)].
- [21] J. Sadeghi, B. Pourhassan, Z. A. Moghaddam, *Interacting Entropy-Corrected Holographic Dark Energy and IR Cut-Off Length*, Int. J. Theor. Phys. **53**, 125 (2014), [[arXiv:1306.2055](#)].
- [22] M. R. Setare, J. Zhang and X. Zhang, *Statefinder diagnosis in a non-flat universe and the holographic model of dark energy*, JCAP **0703**, 007 (2007), [[arXiv:gr-qc/0611084](#)].
- [23] E. N. Saridakis, *Holographic Dark Energy in Braneworld Models with Moving Branes and the $w=-1$ Crossing*, JCAP **0804**, 020 (2008), [[arXiv:0712.2672](#)].
- [24] A. P. Billyard and A. A. Coley, *Interactions in scalar field cosmology*, Phys. Rev. D **61**, 083503 (2000), [[arXiv:gr-qc/astro-ph/9908224](#)].
- [25] A. Nunes, J.P. Mimoso and T.C. Charters, *Scaling solutions from interacting fluids*, Phys. Rev. D **63** (2001) 083506, [[arXiv:gr-qc/0011073](#)].

- [26] G. R. Farrar and P. J. E. Peebles, *Interacting dark matter and dark energy*, Astrophys. J. **604**, 1 (2004), [[arXiv:gr-qc/astro-ph/0307316](#)].
- [27] J.D. Barrow and T. Clifton, *Cosmologies with energy exchange*, Phys. Rev. D **73** (2006) 103520, [[arXiv:gr-qc/0604063](#)].
- [28] T. Gonzalez, G. Leon and I. Quiros, *Dynamics of quintessence models of dark energy with exponential coupling to dark matter*, Class. Quant. Grav. **23** (2006) 3165, [[arXiv:gr-qc/astro-ph/0702227](#)].
- [29] H. Garcia-Compean, G. Garcia-Jimenez, O. Obregon and C. Ramirez, *Crossing the phantom divide in an interacting generalized Chaplygin gas*, JCAP **0807** (2008) 016, [[arXiv:0710.4283](#)].
- [30] C. G. Boehmer, G. Caldera-Cabral, R. Lazkoz and R. Maartens, *Dynamics of dark energy with a coupling to dark matter*, Phys. Rev. D **78**, 023505 (2008), [[arXiv:0801.1565](#)].
- [31] M. Jamil and M.A. Rashid, *Constraining the coupling constant between dark energy and dark matter*, Eur. Phys. J. C **60** (2009) 141, [[arXiv:0802.1144](#)].
- [32] X. -m. Chen, Y. -g. Gong and E. N. Saridakis, *Phase-space analysis of interacting phantom cosmology*, JCAP **0904**, 001 (2009), [[arXiv:0812.1117](#)].
- [33] M. Khurshudyan, B. Pourhassan, E.O. Kahya, *Interacting two-component fluid models with varying EoS parameter*, Int. J. Geom. Meth. Mod. Phys. **11** (2014) 1450061, [[arXiv:gr-qc/1312.1162](#)].
- [34] A. Y. Kamenshchik, U. Moschella and V. Pasquier, *An alternative to quintessence*, Phys. Lett. B **511**, 265 (2001), [[arXiv:gr-qc/0103004](#)].
- [35] M. C. Bento, O. Bertolami and A. A. Sen, *Generalized Chaplygin gas, accelerated expansion and dark energy matter unification*, Phys. Rev. D **66**, 043507 (2002), [[arXiv:gr-qc/0202064](#)].
- [36] J. D. Barrow, *The deflationary universe: An instability of the de Sitter universe* Phys. Lett. B **180**, 335 (1986).
- [37] J. D. Barrow, *String-driven inflationary and deflationary cosmological models*, Nucl. Phys. B **310**, 743 (1988).
- [38] N. Bilic, G.B. Tupper, and R.D. Viollier, *Unification of dark matter and dark energy: the inhomogeneous Chaplygin gas*, Phys. Lett. B **535** (2002) 17, [[arXiv:astro-ph/0111325](#)].
- [39] H. Saadat and B. Pourhassan, *Effect of Varying Bulk Viscosity on Generalized Chaplygin Gas*, Int. J. Theor. Phys. **53**, 1168 (2014) [[arXiv:1305.6054](#)].
- [40] X-H. Zhai, Y-D. Xu, X-Z. Li, *Viscous generalized Chaplygin gas*, Int. J. Mod. Phys. D **15**, 1151 (2006) [[arXiv:astro-ph/0511814](#)].
- [41] Y. D. Xu *et al.* *Generalized Chaplygin gas model with or without viscosity in the w - w' plane*, Astrophys. Space Sci. **337**, 493 (2012).
- [42] A.R. Amani and B. Pourhassan, *Viscous Generalized Chaplygin gas with Arbitrary α* , Int. J. Theor. Phys. **52**, 1309 (2013).
- [43] H. Saadat and B. Pourhassan, *FRW Bulk Viscous Cosmology with Modified Chaplygin Gas in Flat Space*, Astrophys. Space Sci. **343**, 783 (2013).
- [44] U. Debnath, A. Banerjee, and S. Chakraborty, *Role of modified Chaplygin gas in accelerated*

- universe, *Class. Quant. Grav.* **21**, 5609 (2004), [[arXiv:gr-qc/0411015](#)].
- [45] H. Saadat and B. Pourhassan, *FRW bulk viscous cosmology with modified cosmic Chaplygin gas*, *Astrophys. Space Sci.* **344**, 237 (2013).
 - [46] J. Naji, B. Pourhassan, A.R. Amani, *Effect of shear and bulk viscosities on interacting modified Chaplygin gas cosmology*, *Int. J. Mod. Phys. D* **23**, 1450020 (2013).
 - [47] B. Pourhassan, *Viscous Modified Cosmic Chaplygin Gas Cosmology*, *Int. J. Mod. Phys. D* **22**, 1350061 (2013) [[arXiv:1301.2788](#)].
 - [48] J. Sadeghi, B. Pourhassan, M. Khurshudyan, H. Farahani, *Time-Dependent Density of Modified Cosmic Chaplygin Gas with Cosmological Constant in Non-Flat Universe* *Int. J. Theor. Phys.* **53**, 911 (2014).
 - [49] E. V. Linder, R. J. Scherrer, *Aetherizing Lambda: Barotropic fluids as dark energy*, *Phys. Rev. D* **80**, 023008 (2009) [[arXiv:0811.2797](#)].
 - [50] K. N. Ananda and M. Bruni, *Cosmological dynamics and dark energy with a nonlinear equation of state: A quadratic model*, *Phys. Rev. D* **74**, 023523 (2006) [[arXiv:astro-ph/0512224](#)].
 - [51] P. H. Chavanis, *A cosmological model describing the early inflation, the intermediate decelerating expansion, and the late accelerating expansion by a quadratic equation of state*, *Universe* **1**, 357 (2015) [[arXiv:1309.5784](#)].
 - [52] B. Pourhassan, E.O. Kahya, *FRW cosmology with the extended Chaplygin gas*, *Advances in High Energy Physics* **2014**, 231452 (2014) [[arXiv:1405.0667](#)].
 - [53] E.O. Kahya, M. Khurshudyan, B. Pourhassan, R. Myrzakulov, A. Pasqua, *Higher order corrections of the extended Chaplygin gas cosmology with varying G and Λ* , *The European Physical Journal C* **75** (2015) 43 [[arXiv:1402.2592](#)].
 - [54] B. Pourhassan, E.O. Kahya, *Extended Chaplygin gas model*, *Results in Physics* **4**, 101 (2014).
 - [55] E. O. Kahya, B. Pourhassan, *Observational constraints on the extended Chaplygin gas inflation*, *Astrophys. Space Sci.* **353**, 677 (2014).
 - [56] P. Horava, *Membranes at Quantum Criticality*, *JHEP* **0903**, 020 (2009) [[arXiv:0812.4287](#)].
 - [57] G. Calcagni, *Detailed balance in Horava-Lifshitz gravity*, *Phys. Rev. D* **81**, 044006 (2010) [[arXiv:0905.3740](#)].
 - [58] T. Koivisto, D. F. Mota, *Cosmology and Astrophysical Constraints of Gauss-Bonnet Dark Energy*, *Phys. Lett. B* **644**, 104 (2007) [[arXiv:astro-ph/0606078](#)].
 - [59] U. H. Danielsson, L. Thorlacius, *Black holes in asymptotically Lifshitz spacetime*, *JHEP* **0903**, 070 (2009) [[arXiv:0812.5088](#)].
 - [60] R. G. Cai, L. M. Cao, N. Ohta, *Topological Black Holes in Horava-Lifshitz Gravity*, *Phys. Rev. D* **80**, 024003 (2009) [[arXiv:0904.3670](#)].
 - [61] J. Sadeghi, B. Pourhassan, *Particle acceleration in HoravaLifshitz black holes*, *Eur. Phys. J. C* **72**, 1984 (2012) [[arXiv:1108.4530](#)].
 - [62] M. I. Park, *The Black Hole and Cosmological Solutions in IR modified Horava Gravity*, *JHEP* **0909**, 123 (2009) [[arXiv:0905.4480](#)].
 - [63] M. Botta-Cantcheff, N. Grandi, M. Sturla, *Wormhole solutions to Horava gravity*, *Phys. Rev. D* **82**, 124034 (2010) [[arXiv:0906.0582](#)].

- [64] H. W. Lee, Y. W. Kim, Y. S. Myung, *Extremal black holes in the Horava-Lifshitz gravity*, Eur. Phys. J. C **68**, 255 (2010) [[arXiv:0907.3568](#)].
- [65] A. Wang, Y. Wu, *Thermodynamics and classification of cosmological models in the Horava-Lifshitz theory of gravity*, JCAP **0907**, 012 (2009) [[arXiv:0905.4117](#)].
- [66] R. G. Cai, L. M. Cao, N. Ohta, *Thermodynamics of Black Holes in Horava-Lifshitz Gravity*, Phys. Lett. B **679**, 504 (2009) [[arXiv:0905.0751](#)].
- [67] R. G. Cai, N. Ohta, *Horizon Thermodynamics and Gravitational Field Equations in Horava-Lifshitz Gravity*, Phys. Rev. D **81**, 084061 (2010) [[arXiv:0910.2307](#)].
- [68] J. Sadeghi, K. Jafarzade B. Pourhassan, *Thermodynamical Quantities of Horava-Lifshitz Black Hole*, Int. J. Theor. Phys. **51**, 3891 (2012)
- [69] M. Jamil, E. N. Saridakis, M. R. Setare, *The generalized second law of thermodynamics in Horava-Lifshitz cosmology*, JCAP **1011**, 032 (2010) [[arXiv:1003.0876](#)].
- [70] E. N. Saridakis, *Horava-Lifshitz Dark Energy*, Eur. Phys. J. C **67**, 229 (2010) [[arXiv:0905.3532](#)].
- [71] M. I. Park, *A Test of Horava Gravity: The Dark Energy*, JCAP **1001**, 001 (2010) [[arXiv:0906.4275](#)].
- [72] M. Jamil, E. N. Saridakis, *New agegraphic dark energy in Horava-Lifshitz cosmology*, JCAP **1007**, 028 (2010) [[arXiv:1003.5637](#)].
- [73] G. Calcagni, *Cosmology of the Lifshitz universe*, JHEP **0909**, 112 (2009) [[arXiv:0904.0829](#)].
- [74] E. Kiritsis, G. Kofinas, *Horava-Lifshitz Cosmology*, Nucl. Phys. B **821**, 467 (2009) [[arXiv:0904.1334](#)].
- [75] A. Ali, S. Dutta, E. N. Saridakis, A. A. Sen, *Horava-Lifshitz cosmology with generalized Chaplygin gas*, Gen. Relativ. Gravit. **44**, 657 (2012) [[arXiv:1004.2474](#)].
- [76] B. C. Paul, P. Thakur, A. Saha, *Modified Chaplygin gas in Horava-Lifshitz gravity and constraints on its B parameter*, Phys. Rev. D **85**, 024039 (2012)
- [77] B. C. Paul, P. Thakur, *Observational Constraints on Modified Chaplygin Gas in Horava-Lifshitz Gravity*, Pramana **81**, 691 (2012) [[arXiv:1205.2796](#)].
- [78] M. Khodadi, M. Naderi, *Chaplygin Gas Model with Variable G, Λ in Horava-Lifshitz Cosmology*, Int. J. Theor. Phys. **53** (2014) 3988.
- [79] F. C. Santos, M. L. Bedran, V. Soares, *On the thermodynamic stability of the generalized Chaplygin gas*, Phys. Lett. B **636**, 86 (2006)
- [80] Y. S. Myung, *Thermodynamics of Chaplygin gas*, Astrophys. Space Sci. **335**, 561 (2011)
- [81] K. Karami, S. Ghaffari, M. M. Soltanzadeh, *The generalized second law for the interacting generalized Chaplygin gas model*, Astrophys. Space Sci. **331**, 309 (2011)
- [82] M. Salti, *Thermodynamics of Chaplygin Gas Interacting with Cold Dark Matter*, Int. J. Theor. Phys. **52**, 4583 (2013)
- [83] F. C. Santos, M. L. Bedran, V. Soares, *On the thermodynamic stability of the modified Chaplygin gas*, Phys. Lett. B **646**, 215 (2007)
- [84] B. Kr. Dev Choudhury, J. Saikia, *Some Discussion on Thermodynamical Behaviour of Modified Chaplygin Gas*, [[arXiv:1006.1461](#)].

- [85] S. Bhattacharya, U. Debnath, *Thermodynamics of Modified Chaplygin Gas and Tachyonic Field*, Int. J. Theor. Phys. **51**, 565 (2012)
- [86] U. Debnath, M. Jamil, *Correspondence between DBI-essence and Modified Chaplygin Gas and the Generalized Second Law of Thermodynamics*, Astrophys. Space Sci. **335**, 545 (2011)
- [87] T. Bandyopadhyay, *Thermodynamics of Gauss-Bonnet brane with modified Chaplygin gas*, Astrophys. Space Sci. **341**, 689 (2012)
- [88] C. Charmousis, G. Niz, A. Padilla, P. M. Saffin, *Strong coupling in Horava gravity*, JHEP **0908**, 070 (2009) [[arXiv:0905.2579](#)].
- [89] P. Horava, *Membranes at Quantum Criticality*, Phys. Rev. D **79**, 084008 (2009) [[arXiv:0901.3775](#)].
- [90] S. Dutta, E. N. Saridakis, *Observational constraints on Horava-Lifshitz cosmology*, JCAP **1001**, 013 (2010) [[arXiv:0911.1435](#)].
- [91] P. Thakur, S. Ghose, B. C. Paul, *Modified Chaplygin gas and constraints on its B parameter from cold dark matter and unified dark matter energy cosmological models*, Mon. Not. R. Astron. Soc. **397**, 1935 (2009) [[arXiv:0905.2281](#)].
- [92] E. Komatsu, et al., *Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation*, Astrophys. J. Suppl. **192**, 18 (2011) [[arXiv:1001.4538](#)].
- [93] R. Bean, A. Melchiorri, *Current constraints on the dark energy equation of state*, Phys. Rev. D **65**, 041302 (2002) [[arXiv:astro-ph/0110472](#)].
- [94] M. Chevallier and D. Polarski, *Accelerating Universes with Scaling Dark Matter* International Journal of Modern Physics D **10**, 213 (2001)
- [95] E. V. Linder, *Exploring the Expansion History of the Universe* Phys. Rev. Lett. **90**, 091301 (2003)
- [96] J. A. Vazquez, M. Bridges, M. P. Hobson, A. N. Lasenby, *Reconstruction of the Dark Energy equation of state*, JCAP **1209**, 020 (2012) [[arXiv:1205.0847](#)].
- [97] O. Avsajanishvili, L. Samushia, N. A. Arkhipova, T. Kahnishvili, *Testing Dark Energy Models through Large Scale Structure*, [[arXiv:1511.09317](#)].
- [98] J. Sadeghi, B. Pourhassan, and F. Rahimi, *Logarithmic corrections of charged hairy black holes in $(2 + 1)$ dimensions*, Can. J. Phys. **92**, 1638 (2014)